Title Lecture 9

Date 2023 · 11 · 29

Review

$R_{TM} = \{``M": ``M" \text{ is a TM } L(M) \text{ regular}\}$

sol. $H \leq \bar{R}_{TM}$

$M_2 \leftarrow M_1$

M_2 = on input "M" "w"

1. construct TM "M'" (i) run M on w

(ii) run U on x

$L(M') = \emptyset$ (M not halt) regular

$L(M') = H$ (M halts) non-regular

so $H \leq \bar{R}_{TM}$

- Rice's Theorem

对于 {M : M is a TM with L(M) has prosperty P}

↳ $\{M : M \text{ is a TM with } L(M) \in \mathcal{L}(P)\}$

若 $\mathcal{L}(P)$ 为 $\mathcal{L}_{re}$ = {L : L is recursive enumerable} 的非空真子集

↓ 有 L 满足 P，有 L 不满足

上述 language 不可判定

证明: sol. ↓ $H \leq R(P)$

1. $\emptyset \notin \mathcal{L}(P)$ $\exists L \in \mathcal{L}(P) \Rightarrow \exists M_A$ 半判定 L

M_H = on input "M" "w"

1. 构造 M^* = on input x (1) run M on w (2) run M_A on L

$L(M^*) = \begin{cases} L(M_A) = A & \text{halt} \quad A \in \mathcal{L}(P) \\ \emptyset & \text{not halt} \end{cases}$

2. run "M_R" on "M^*" 3. return M_R result.

2. $\emptyset \in \mathcal{L}(P)$，证明 $H \leq \overline{\mathcal{L}(P)}$

{ 证明可判定： A ≤ 可判定问题
证明不可判定：不可判定问题 ≤ A

证明半判定

A = {"M" : M is a TM halts on some input}

sol. 列出所有 string，并行运行所有 string

在某一时刻，运行 $S_1 - S_i$ 到 i 步，若有 S_k 停机，则接受；若无停机，下一时刻运行 $S_1 - S_{i+1}$ 到 i+1 步.

M_A = on input "M" → for i = 1, 2, 3, ...
for s = $S_1, S_2, \ldots$
run M on s to i steps
if halts accept.

证明不可判定

可用归约

若 A 及 $\bar{A}$ 均可半判定，则 A 可判定

$\exists M_1$ 半判定 A M_2 半判定 $\bar{A}$

M_3 decide A

1. run M_1, M_2 并行
2. if M_1 halts accept A
3. if M_2 halts reject $\bar{A}$

{ H 半判定 → $\bar{H}$ 不可半判定
H 不可判定

closure property

∪ ∩ ‾ * ∘ → 均可保持 recursive 的性质

↳ 例举所有组合

∪ ∩ * ∘ → 保持 recursively enumerable 性质.

（补集不闭包）

Title

Enumerator

output state ↑

a TM enumerate a language L, for some q,

$L = \{w : (s, \triangleright \sqcup) \vdash_M^* (q, \triangleright \sqcup w)\}$ 输出 w

↳ 图灵可枚举.

L is turing enumerable iff 可半判定、

证明：若 L 有限，易得，下证 L 无限.

sol. $\exists M$ enumerates $L \Leftrightarrow M'$ semidecides L

① M' = on input x 1. run M to enumerate L

左 → 右 2. M output w 3. if $w == x$ halt

② M_A = on input "M" → for $i = 1, 2, 3, \cdots$

右 → 左 (与此前的证明类似) for $s = s_1, s_2, \cdots$

run M on s to i steps

可能重复.乱序 ← if halts output s

- M lexicographically enumerate L if whenever

$(q, \triangleright \sqcup w) \vdash_M^+ (q, \triangleright \sqcup w')$ $\vdash_M^+$ 表示至少走一步

w' after w in lexicographically order

L is lexicographically enumerable iff recursive (字典序)

证明: ① 左推右 M decides L 1. enumerate all strings

2. for each s run M on s

②右推左 3. if M accepts then accepts

M' = on input x 1. run M to enumerate all strings (字典序

2. every time output w 3. if $w == x$ accept

4. reject x